I was invited to the Math Olympiad Summer Program (MOP) in the 10th grade. I went to MOP certain that I must really be good at math. In my five weeks at MOP, I encountered over sixty problems on various tests. I didn’t solve a single one. That’s right - I was 0-for-60+. I came away no longer confident that I was good at math. I assumed that most of the other kids did better at MOP because they knew more tricks than I did. My formula sheets were pretty thorough, but perhaps they were missing something. By the end of MOP, I had learned a somewhat unsettling truth. The others knew fewer tricks than I did, not more. They didn’t even have formula sheets!

At another contest later that summer, a younger student, Alex, from another school asked me for my formula sheets. In my local and state circles, students’ formula sheets were the source of knowledge, the source of power that fueled the top students and the top schools. They were studied, memorized, revered. But most of all, they were not shared. But when Alex asked for my formula sheets I remembered my experience at MOP and I realized that formula sheets are not really math. Memorizing formulas is no more mathematics than memorizing dates is history or memorizing spelling words is literature. I gave him the formula sheets. (Alex must later have learned also that the formula sheets were fool’s gold - he became a Rhodes scholar.)

The difference between MOP and many of these state and local contests I participated in was the difference between problem solving and what many people call mathematics. For these people, math is a series of tricks to use on a series of specific problems. Trick A is for Problem A, Trick B for Problem B, and so on. In this vein, school can become a routine of ‘learn tricks for a week - use tricks on a test - forget most tricks quickly.’ The tricks get forgotten quickly primarily because there are so many of them, and also because the students don’t see how these ‘tricks’ are just extensions of a few basic principles.

I had painfully learned at MOP that true mathematics is not a process of memorizing formulas and applying them to problems tailor-made for those formulas. Instead, the successful mathematician possesses fewer tools, but knows how to apply them to a much broader range of problems. We use the term “problem solving” to distinguish this approach to mathematics from the ‘memorize-use-forget’ approach.

After MOP I relearned math throughout high school. I was unaware that I was learning much more. When I got to Princeton I enrolled in organic chemistry. There were over 200 students in the course, and we quickly separated into two groups. One group understood that all we would be taught could largely be derived from a very small number of basic principles. We loved the class - it was a year long exploration of where these fundamental concepts could take us. The other, much larger, group saw each new destination not as the result of a path from the building blocks, but as yet another place whose coordinates had to be memorized if ever they were to visit again. Almost to a student, the difference between those in the happy group and those in the struggling group was how they learned mathematics. The class seemingly involved no math at all, but those who took a memorization approach to math were doomed to do it again in chemistry. The skills the problem solvers developed in math transferred, and these students flourished.

We use math to teach problem solving because it is the most fundamental logical discipline. Not only is it the foundation upon which sciences are built, it is the clearest way to learn and understand how to develop a rigorous logical argument. There are no loopholes, there are no half-truths. The
What is Problem Solving?
by Richard Rusczyk
Reprinted from www.artofproblemsolving.com

language of mathematics is precise, as is ‘right’ and ‘wrong’ (or ‘proven’ and ‘unproven’). Success and failure are immediate and indisputable; there isn’t room for subjectivity. This is not to say that those who cannot do math cannot solve problems. There are many paths to strong problem solving skills. Mathematics is the shortest.

Problem solving is crucial in mathematics education because it transcends mathematics. By developing problem solving skills, we learn not only how to tackle math problems, but also how to logically work our way through any problems we may face. The memorizer can only solve problems he has encountered already, but the problem solver can solve problems she’s never seen before. The problem solver is flexible; she can diversify. Above all, she can create.
Creative Thinking

Claude Shannon at Bell Lab.
March 20, 1952

Up to 100% of the amount of ideas produced, useful good ideas produced by these signals, these are supposed to be arranged in order of increasing ability. At producing ideas, we find a curve something like this. Consider the number of curves produced here - going up to enormous height here.

A very small percentage of the population produces the greatest proportion of the important ideas. This is akin to an idea presented by an English mathematician, Turing, that the human brain is something like a piece of uranium. The human brain, if it is below the critical lap and you shoot one neutron into it, additional more would be produced by impact. It leads to an extremely explosive of the issue, increase the size of the uranium. Turing says this is something like ideas in the human brain. There are some people if you shoot one idea into the brain, you will get a half an idea out. There are other people who are beyond this point at which they produce two ideas for each idea sent in. those are the people beyond the knee of the curve. I don't want to sound egotistical here, I don't think that I am beyond the knee of this curve and I don't know anyone who is. I do know some people that were. I think, for example, that anyone will agree that Isaac Newton would be well on the top of this curve. When you think that at the age of 25 he had produced enough science, physics and mathematics to make 10 or 20 men famous - he produced binomial theorem, differential and integral calculus, laws of gravitation, laws of motion, decomposition of white light, and so on. Now what is it that shoots one up to this part of the curve? What are the basic requirements? I think we could set down three things that are fairly necessary for scientific research or for any sort of inventing or mathematics or physics or anything along that line. I don't think a person can get along without any one of these three.

The first one is obvious - training and experience. You don't expect a lawyer, however bright he may be, to give you a new theory of physics these days or mathematics or engineering.

The second thing is a certain amount of intelligence or talent. In other words, you have to have an IQ that is fairly high to do good research work. I don't think that there is any good engineer or scientist that can get along on an IQ of 100, which is the average for human beings. In other words, he has to have an IQ higher than that. Everyone in this room is considerably above that. This, we might say, is a matter of environment; intelligence is a matter of heredity.

Those two I don't think are sufficient. I think there is a third constituent here, a third component which is the one that makes an Einstein or an Isaac Newton. For want of a better word, we will call it motivation. In other words, you have to have some kind of a drive, some kind of a desire to find out the answer, a desire to find out what makes things tick. If you don't have that, you may have all the training and intelligence in the world, you don't have questions and you won't just find answers. This is a hard thing to put your finger on. It is a matter of temperament probably; that is, a matter of probably early training, early childhood experiences, whether you will motivate in the direction of scientific research. I think that at a superficial level, it is blended use of several things. This is not any attempt at a deep analysis at all, but my feeling is that a good scientist has a great deal of what we can call curiosity. I won't go any deeper into it than that. He wants to know the answers. He's just curious how things tick and he wants to know the answers to questions; and if he sees things, he wants to raise questions and he wants to know the answers to those.

Then there's the idea of dissatisfaction. By this I don't mean a pessimistic dissatisfaction of the world - we don't like the way things are - I mean a constructive dissatisfaction. The idea could be expressed in the words, °∞This is OK, but I think things could be done better. I think there is a neater way to do this. I think things could be improved a little. In other words, there is continually a slight irritation when things don’t look quite right; and I think that dissatisfaction in present days is a key driving force in good scientists.

And another thing I°Ød put down here is the pleasure in seeing net results or methods of arriving at results needed, designs of engineers, equipment, and so on. I get a big bang myself out of providing a theorem. If I’ve been trying to prove a mathematical theorem for a week or so and I finally find the solution, I get a big bang out of it. And I get a big kick out of seeing a clever way of doing some engineering problem, a clever design for a circuit which uses a very small amount of equipment and gets apparently a great deal of result out of it. I think so far as motivation is concerned, it is maybe a little like Fats Waller said about swing music -
"either you got it or you ain’t." if you ain’t got it, you probably shouldn’t be doing research work if you don’t want to know that kind of answer. Although people without this kind of motivation might be very successful in other fields, the research man should probably have an extremely strong drive to want to find out the answers, so strong a drive that he doesn’t care whether it is 5 o’clock - he is willing to work all night to find out the answers and all weekend if necessary. Well now, this is all well and good, but supposing a person has these three properties to a sufficient extent to be useful, are there any tricks, any gimmicks that he can apply to thinking that will actually aid in creative work, in getting the answers in research work, in general, in finding answers to problems? I think there are, and I think they can be catalogued to an certain extent. You can make quite a list of them and I think they would be very useful if one did that, so I am going to give a few of them which I have thought up or which people have suggested to me. And I think if one consciously applied these to various problems you had to solve, in many cases you’d find solutions quicker than you would normally or in cases where you might not find it at all. I thing that good research workers apply these things unconsciously; that is, they do these things automatically and if they were brought forth into the conscious thinking that here’s a situation where I would try this method of approach that would probably get there faster, although I can’t document this statement.

**The first one that I might speak of is the idea of simplification.** Suppose that you are given a problem to solve, I don’t care what kind of a problem - a machine to design, or a physical theory to develop, or a mathematical theorem to prove, or something of that kind - probably a very powerful approach to this is to attempt to eliminate everything from the problem except the essentials; that is, cut it down to size. Almost every problem that you come across is befuddled with all kinds of extraneous data of one sort or another; and if you can bring this problem down into the main issues, you can see more clearly what you’re trying to do and perhaps find a solution. Now, in so doing, you may have stripped away the problem that you’re after. You may have simplified it to a point that it doesn’t even resemble the problem that you started with; but very often if you can solve this simple problem, you can add refinements to the solution of this until you get back to the solution of the one you started with.

**A very similar device is seeking similar known problems.** I think I could illustrate this schematically in this way. You have a problem P here and there is a solution S which you do not know yet perhaps over here. If you have experience in the field represented, that you are working in, you may perhaps know of a somewhat similar problem, call it P’, which has already been solved and which has a solution, S’, all you need to do - all you may have to do is find the analogy from P’ here to P and the same analogy from S’ to S in order to get back to the solution of the given problem. This is the reason why experience in a field is so important that if you are experienced in a field, you will know thousands of problems that have been solved. Your mental matrix will be filled with P’s and S’s unconnected here and you can find one which is tolerably close to the P that you are trying to solve and go over to the corresponding S’ in order to go back to the S you’re after. It seems to be much easier to make two small jumps than the one big jump in any kind of mental thinking.

**Another approach for a given problem is to try to restate it in just as many different forms as you can.** Change the words. Change the viewpoint. Look at it from every possible angle. After you’ve done that, you can try to look at it from several angles at the same time and perhaps you can get an insight into the real basic issues of the problem, so that you can correlate the important factors and come out with the solution. It’s difficult really to do this, but it is important that you do. If you don’t, it is very easy to get into ruts of mental thinking. You start with a problem here and you go around a circle here and if you could only get over to this point, perhaps you would see your way clear; but you can’t break loose from certain mental blocks which are holding you in certain ways of looking at a problem. That is the reason why very frequently someone who is quite green to a problem will sometimes come in and look at it and find the solution like that, while you have been laboring for months over it. You’ve got set into some ruts here of mental thinking and someone who is quite green to a problem will sometimes come in and look at it and find the solution like that, while you have been laboring for months over it. You’ve got set into some ruts here of mental thinking and someone else comes in and sees it from a fresh viewpoint.

**Another mental gimmick for aid in research work, I think, is the idea of generalization.** This is very powerful in mathematical research. The typical mathematical theory developed in the following way to prove a very isolated, special result, particular theorem - someone always will come along and start generalization it. He will leave it where it was in two dimensions before he will do it in N dimensions; or if it was in some kind of algebra, he will work in a general algebraic field; if it was in the field of real numbers, he will change it to a general algebraic field or something of that sort. This is actually quite easy to do if you only remember to do it. If the minute you’ve found an answer to something, the next thing to do is to ask yourself if you can generalize this anymore - can I make the same, make a broader statement which includes more - there, I think, in terms of engineering, the same thing should be kept in mind. As you see, if somebody comes along with a clever way of doing something, one should ask oneself °∞Can I apply the same principle in more
Next one I might mention is the idea of structural analysis of a problem. Suppose you have your problem here and a solution here. You may have two big a jump to take. What you can try to do is to break down that jump into a large number of small jumps. If this were a set of mathematical axioms and this were a theorem or conclusion that you were trying to prove, it might be too much for me try to prove this thing in one fell swoop. But perhaps I can visualize a number of subsidiary theorems or propositions such that if I could prove those, in turn I would eventually arrive at this solution. In other words, I set up some path through this domain with a set of subsidiary solutions, 1, 2, 3, 4, and so on, and attempt to prove this on the basis of that and then this one the basis of these which I have proved until eventually I arrive at the path S. Many proofs in mathematics have been actually found by extremely roundabout processes. A man starts to prove this theorem and he finds that he wanders all over the map. He starts off and prove a good many results which don’t seem to be leading anywhere and then eventually ends up by the back door on the solution of the given problem; and very often when that’s done, when you’ve found your solution, it may be very easy to simplify; that is, to see at one stage that you may have short-cutted across here and you could see that you might have short-cutted across there. The same thing is true in design work. If you can design a way of doing something which is obviously clumsy and cumbersome, uses too much equipment; but after you’ve really got something you can get a grip on, something you can hang on to, you can start cutting out components and seeing some parts were really superfluous. You really didn’t need them in the first place.

Now one other thing I would like to bring out which I run across quite frequently in mathematical work is the idea of inversion of the problem. You are trying to obtain the solution S on the basis of the premises P and then you can’t do it. Well, turn the problem over supposing that S were the given proposition, the given axioms, or the given numbers in the problem and what you are trying to obtain is P. Just imagine that that were the case. Then you will find that it is relatively easy to solve the problem in that direction. You find a fairly direct route. If so, it’s often possible to invent it in small batches. In other words, you’ve got a path marked out here - there you got relays you sent this way. You can see how to invert these things in small stages and perhaps three or four only difficult steps in the proof.

Now I think the same thing can happen in design work. Sometimes I have had the experience of designing computing machines of various sorts in which I wanted to compute certain numbers out of certain given quantities. This happened to be a machine that played the game of nim and it turned out that it seemed to be quite difficult. If took quite a number of relays to do this particular calculation although it could be done. But then I got the idea that if I inverted the problem, it would have been very easy to do - if the given and required results had been interchanged; and that idea led to a way of doing it which was far simpler than the first design. The way of doing it was doing it by feedback; that is, you start with the required result and run it back until - run it through its value until it matches the given input. So the machine itself was worked backward putting range S over the numbers until it had the number that you actually had and, at that point, until it reached the number such that P shows you the correct way. Well, now the solution for this philosophy which is probably very boring to most of you. I’d like now to show you this machine which I brought along and go into one or two of the problems which were connected with the design of that because I think they illustrate some of these things I’ve been talking about. In order to see this, you’ll have to come up around it; so, I wonder whether you will all come up around the table now.
Polya’s Problem Solving Techniques

In 1945 George Polya published the book *How To Solve It* which quickly became his most prized publication. It sold over one million copies and has been translated into 17 languages. In this book he identifies four basic principles of problem solving.

**Polya’s First Principle: Understand the problem**

This seems so obvious that it is often not even mentioned, yet students are often stymied in their efforts to solve problems simply because they don’t understand it fully, or even in part. Polya taught teachers to ask students questions such as:

- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?

**Polya’s Second Principle: Devise a plan**

Polya mentions that there are many reasonable ways to solve problems. The skill at choosing an appropriate strategy is best learned by solving many problems. You will find choosing a strategy increasingly easy. A partial list of strategies is included:

- Guess and check
- Make an orderly list
- Eliminate possibilities
- Use symmetry
- Consider special cases
- Use direct reasoning
- Solve an equation
- Look for a pattern
- Draw a picture
- Solve a simpler problem
- Use a model
- Work backwards
- Use a formula
- Be ingenious
Polya’s Third Principle: Carry out the plan

This step is usually easier than devising the plan. In general, all you need is care and patience, given that you have the necessary skills. Persist with the plan that you have chosen. If it continues not to work discard it and choose another. Don’t be misled, this is how mathematics is done, even by professionals.

Polya’s Fourth Principle: Look back

Polya mentions that much can be gained by taking the time to reflect and look back at what you have done, what worked, and what didn’t. Doing this will enable you to predict what strategy to use to solve future problems.

1. UNDERSTAND THE PROBLEM

- **First.** You have to *understand* the problem.
- What is the unknown? What are the data? What is the condition?
- Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
- Draw a figure. Introduce suitable notation.
- Separate the various parts of the condition. Can you write them down?

2. DEVISING A PLAN

- **Second.** Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.
- Have you seen it before? Or have you seen the same problem in a slightly different form?
- *Do you know a related problem?* Do you know a theorem that could be useful?
- *Look at the unknown!* Try to think of a familiar problem having the same or a similar unknown.
- *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
- Could you restate the problem? Could you restate it still differently? Go back to definitions.
- If you cannot solve the proposed problem, try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?
3. CARRYING OUT THE PLAN

- **Third.** *Carry out* your plan.
- Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?

4. LOOKING BACK

- **Fourth.** *Examine* the solution obtained.
- Can you *check the result*? Can you check the argument?
- Can you derive the solution differently? Can you see it at a glance?
- Can you use the result, or the method, for some other problem?
How to Think

A View from Ed Boyden

Managing brain resources in an age of complexity.

November 13, 2007

When I applied for my faculty job at the MIT Media Lab, I had to write a teaching statement. One of the things I proposed was to teach a class called “How to Think,” which would focus on how to be creative, thoughtful, and powerful in a world where problems are extremely complex, targets are continuously moving, and our brains often seem like nodes of enormous networks that constantly reconfigure. In the process of thinking about this, I composed 10 rules, which I sometimes share with students. I’ve listed them here, followed by some practical advice on implementation.

1. **Synthesize new ideas constantly.** Never read passively. Annotate, model, think, and synthesize while you read, even when you’re reading what you conceive to be introductory stuff. That way, you will always aim towards understanding things at a resolution fine enough for you to be creative.

2. **Learn how to learn (rapidly).** One of the most important talents for the 21st century is the ability to learn almost anything instantly, so cultivate this talent. Be able to rapidly prototype ideas. Know how your brain works. (I often need a 20-minute power nap after loading a lot into my brain, followed by half a cup of coffee. Knowing how my brain operates enables me to use it well.)

3. **Work backward from your goal.** Or else you may never get there. If you work forward, you may invent something profound—or you might not. If you work backward, then you have at least directed your efforts at something important to you.

4. **Always have a long-term plan.** Even if you change it every day. The act of making the plan alone is worth it. And even if you revise it often, you’re guaranteed to be learning something.

5. **Make contingency maps.** Draw all the things you need to do on a big piece of paper, and find out which things depend on other things. Then, find the things that are not dependent on anything but have the most dependents, and finish them first.

6. **Collaborate.**

7. **Make your mistakes quickly.** You may mess things up on the first try, but do it fast, and then move on. Document what led to the error so that you learn what to recognize, and then move on. Get the mistakes out of the way. As Shakespeare put it, “Our doubts are traitors, and make us lose the good we oft might win, by fearing to attempt.”

8. As you develop skills, write up best-practices protocols. That way, when you return to something you’ve done, you can make it routine. Instinctualize conscious control.

9. **Document everything obsessively.** If you don’t record it, it may never have an impact on the world. Much of creativity is learning how to see things properly. Most profound scientific discoveries are surprises. But if you don’t document and digest every observation and learn to trust your eyes, then you will not know when you have seen a surprise.
10. **Keep it simple.** If it looks like something hard to engineer, it probably is. If you can spend two days thinking of ways to make it 10 times simpler, do it. It will work better, be more reliable, and have a bigger impact on the world. And learn, if only to know what has failed before. Remember the old saying, “Six months in the lab can save an afternoon in the library.”

Two practical notes. The first is in the arena of time management. I really like what I call **logarithmic time planning**, in which events that are close at hand are scheduled with finer resolution than events that are far off. For example, things that happen tomorrow should be scheduled down to the minute, things that happen next week should be scheduled down to the hour, and things that happen next year should be scheduled down to the day. Why do all calendar programs force you to pick the exact minute something happens when you are trying to schedule it a year out? I just use a word processor to schedule all my events, tasks, and commitments, with resolution fading away the farther I look into the future. (It would be nice, though, to have a software tool that would gently help you make the schedule higher-resolution as time passes…)

The second practical note: I find it really useful to write and draw while talking with someone, composing **conversation summaries** on pieces of paper or pages of notepads. I often use plenty of color annotation to highlight salient points. At the end of the conversation, I digitally photograph the piece of paper so that I capture the entire flow of the conversation and the thoughts that emerged. The person I’ve conversed with usually gets to keep the original piece of paper, and the digital photograph is uploaded to my computer for keyword tagging and archiving. This way I can call up all the images, sketches, ideas, references, and action items from a brief note that I took during a five-minute meeting at a coffee shop years ago—at a touch, on my laptop. With 10-megapixel cameras costing just over $100, you can easily capture a dozen full pages in a single shot, in just a second.

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The key to success in everything is effective thinking. Most of us assume that effective thinking is an inborn talent. But the fact is all of us can learn them and use them. In the book *The 5 Elements of Effective Thinking* – Edward B. Burger and Michael Starbird shows how anyone can think effectively by using 5 key elements.

> I know quite certainly that I myself have no special talent. Curiosity, Obsession and dogged endurance, combined with self-criticism, have brought me to my ideas – Albert Einstein

### 1. Understand Deeply

When you learn anything, **go for depth** and make it **rock solid**. Any concept that you are trying to master is a combination of simple core ideas. Identify the core ideas and learn them deeply. What is deep understanding? Imagine that you wanted to understand basic economics. List down its core components – Maximize profit, free markets; supply and demand. This is not the complete list. But it is a decent one to get started with. Now you want to learn about the equilibrium of supply and demand. Excerpt from the book.

> First, I need to understand what graphs of the supply and demand curves mean. The horizontal axis is the quantity and the vertical axis is the price; so I see why the demand graph curves down to the right and the supply graph curves up to the right. I think that equilibrium is the point of intersection of those two graphs. But if the quantity level is to the left of that intersection, then the price for demand is higher than the price for supply. I don’t know what that means. (Note that this student successfully identified a lack of understanding of a basic idea, namely, what the supply and demand graphs represent. He now knows what he should work on first. A firm understanding of that basic idea will allow him to progress further and faster in the future.)

Few years back I came across an article on What is Problem Solving written by Richard Rusczyk. Richard is the founder of *Art of Problem Solving*. The article was an eye opener for me. Excerpt from the article.

> I was invited to the Math Olympiad Summer Program (MOP) in the 10th grade. I went to MOP certain that I must really be good at math. In my five weeks at MOP, I encountered over sixty problems on various tests. I didn’t solve a single one. That’s right – I was 0-for-60+. I came away no longer confident that I was good at math. I assumed that most of the other kids did better at MOP because they knew more tricks than I did. My formula sheets were pretty thorough, but perhaps they were missing something. By the end of MOP, I had learned a somewhat unsettling truth. The others knew fewer tricks than I did, not more. They didn’t even have formula sheets! … The difference between MOP and many of these state and local contests I participated in was the difference between problem solving and what many people call mathematics. For these people, math is a series of tricks to use on a series of specific problems. Trick A is for Problem A, Trick B for Problem B, and so on. In this vein, school can become a routine of ‘learn tricks for a week – use tricks on a test – forget most tricks quickly.’ The tricks get forgotten quickly primarily because there are so many of them, and also because the students don’t see how these ‘tricks’ are just extensions of a few basic principles.
The key is to identify the core ideas and learn them deeply. Be brutally honest with you. If you do not understand go back to the core concepts again and again. Rag the concept. You will get it at some point.

Memorizing is not deep learning.

Video – Michael Starbird on Understanding Deeply

2. Make Mistakes

Mistakes are great teachers. They highlight unforeseen opportunities and holes in our understanding.

I’ve missed more than 9,000 shots in my career. I’ve lost almost 300 games. 26 times, I’ve been trusted to take the game winning shot and missed. I’ve failed over and over and over again in my life. And that is why I succeed. – Michael Jordan

One day author Starbird gave a difficult problem in mathematics on infinity to his class students. He was aware that this was a hard question and beyond the reach of his students. He picked up Mary to answer this question. Mary hated Mathematics. The conversation between him and Mary :

Mary - I don't want to say it, because I know it's wrong.
Starbird - I'm sure it's wrong, but I still want to hear it.
Mary - Gave the answer.
Starbird - Congratulated her, "You're right - your solution is wrong!"
Mary - Smiled.
Starbird - Tell me just one thing that is wrong in the answer.
Mary - Pointed out something that missing from her answer.
Starbird - Great! Now how do you fix that defect.
Mary - After some thought she fixed the defect.
Starbird - Great! Is the solution correct now?
Mary - No.
Starbird - Tell me just one thing that is wrong in the answer.
Mary - Identified another mistake and corrected it.

After several iterations Mary was able to solve the problem. The solution she discovered was creatively different from the standard answer found in the textbooks. The key take away is that you cannot come out with a correct solution on the first attempt. Start with a probable solution (hypothesis) and keep on correcting the mistakes until you arrive at the correct solution. Thomas Edison was famous for using this approach for inventions.

Try something: see what’s wrong; learn from the defect; try again. When he said that invention is 1% inspiration and 99% perspiration, the perspiration was the process of incrementally making mistakes and learning from them to make the next attempts apt to be closer to right. When Edison was asked how he felt about his countless failed attempts at making a lightbulb, he replied, “I have not failed. I’ve just found 10,000 ways that won’t work.”

Sometime your attempt would have failed to solve one issue. But it could be the best solution for another issue. A
A bad solution to one problem might be the best solution for another. This is what happened in the company 3M.

In 1970, 3M scientist Spencer Silver was working hard to create an even stronger adhesive. His creation was a resounding failure. In fact, the bond was actually weaker than other 3M products of the day – it was so weak it could be stuck to objects and then easily lifted off them without a trace. 3M did not fire him. Wise move, since four years later, when 3M scientist Arthur Fry was trying to devise a way of placing bookmarks in his hymnal so they would neither fall out nor damage the page, he recalled his colleague’s weak mixture. Fry coated part of his bookmarks with Silver’s super weak adhesive and thus accidentally gave birth to one of 3M’s most lucrative products: the Post-it note.

Video – Michael Starbird on Make Mistakes

3. Raise Questions

If you want to deepen your understanding you need to raise questions. Do not be afraid to show your ignorance. If you do not understand ask. Socrates a great philosopher who is known for asking great questions.

He challenged his students, friends, and even enemies to make new discoveries by asking them uncomfortable, core questions. You would certainly be astonishingly successful if you had your very own personal Socrates with you at all times, prodding you with the right leading questions. In fact, such as 24/7 Socrates is possible, because you can generate your own questions that challenge your own assumptions and lead to insights. You can become your own Socrates.

On January 28, 1986 space shuttle Challenger exploded. A presidential committee was formed to probe the explosion. Richard Feynman was one of the committee members. The night before the launch the air temperature was very low. During the launch the videos showed that there was a misalignment between the two parts of the solid rocket boaster. Hence the probe moved towards investigating the O-ring seals used between the segments of the boosters. The engineering details are extremely complex and involves chemistry, physics, and mechanics. What did Feynman do? He asked the question.

What if we just test the elasticity of a cooled O-ring?

Video on Feynman solving the puzzle

In fact, he conducted a simple demonstration live on the televised broadcast of the investigation. He took on of NASA’s O-rings, clamped it down with a little C-clamp, and submerged it in a paper cup filled with ice water. When he removed the C-clamp, the entire coast-to-coast audience could see that the cold rubber did not return to its previous round shape. The miserable mystery was solved.

Remember to
Never pretend to know more than you do. Don’t build on ambiguity and ignorance. When you don’t know something, admit it as quickly as possible and immediately take action – ask a question.

Video – Michael Starbird on Raise Questions

4. Follow the flow of ideas

To truly understand a concept, discover how it evolved from existing simpler concepts. Recognizing present reality is a moment in a continuing evolution makes your understanding fit into a more coherent structure. Charlie Munger calls this coherent structure as Latticework of mental models. Calculus is the mathematical study of change. It has truly changed the world. But this did not happen on the day it was discovered.

During the past three hundred years, calculus has been applied to mechanics, to the motion of the planets, to electricity and magnetism, to fluid flow, to biology, to economics, as well as to countless other areas. Calculus may hold a world’s record for how far an idea can be pushed. Leibniz published the first article on calculus in 1684, an essay that was a mere 6 pages long. Newton and Leibniz would surely be astounded to learn that today’s introductory calculus textbook contains over 1,300 pages.

The textbook introduces two fundamental ideas which is 6 pages long. The remaining 1,294 pages consists of variations arising from the two fundamental concepts. The key is to master the core and see how one idea leads to another.

As you are learning a topic, ask yourself what previous knowledge and what strategy of extending previous ideas make the new idea clear, intuitive, and a natural extension.

You cannot discover everything on your own. You need to use the existing ideas and improve it.

Thomas Edison was supremely successful at inventing product after product, exploiting the maxim that every new idea has utility beyond its original intent, for he wrote, “I start where the last man left off.” more poignantly he noted that “many of life’s failures are people who did not realize how close they were to success when they gave up.”

Video – Michael Starbird on Follow the flow of ideas

5. Change

By mastering the first four elements, you can change the way you think and learn.

You simply need to shrug off perhaps a lifetime’s habit of accepting a relatively superficial level of understanding and start understanding more deeply. You simply need to let go of the constraining forces in your life and let yourself fail on the road to success. You simply need to question all the issues you have taken for granted all those years. You simply need to see every aspect of your world
Learning is a life long journey.

Each of us remains a work-in-progress – always evolving, every changing – and that’s Quintessential living.

Video – Michael Starbird on Change